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Intensity and coherence function in a field due to a slit illuminated by a line source

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Abstract. The intensity and degree of coherence in a field due to a slit illuminated by a line source are studied. The propagation of the mutual coherence of a polychromatic light beam is used to obtain the intensity and the degree of coherence due to the slit at points distant from those at the slit. The variation of the intensity and of the degree of coherence with different variables are studied.

1. Introduction

In interference theory it is well known that the intensities and amplitudes of the interfering beams are added up for the cases of incoherent and coherent sources respectively. These two cases are mathematically the most simple and have been dealt with extensively in textbooks. But, in practice, no sources fall into either of these categories. Masers are known (Mandel 1961) to be partially coherent. The thermal sources which are incoherent, to a good approximation, are seldom used directly. We always obtain an experimental source by limiting the radiation of a parent source through a slit or else by concentrating its light on the slit by an optical system. If we keep the slit at a distance D from the parent thermal source, of area S, the slit will contain coherent patches extending over an area of the order of $\lambda^2 D^2/4S$ (Mandel 1961), where λ is the mean wavelength. By coherent patches, we mean an area over which the fields at any two points have appreciable correlation. Even when the light is concentrated on the slit by an optical system, the slit contains coherent patches. This is because, in all practical cases, a point of the source does not form a point image. Obviously, the area of the coherent patches in this case is the same as the area of the image of any point. Investigation of partial coherent sources is thus important not only from the theoretical but also from the experimental point of view.

Extensive work (for a complete bibliography see Mandel and Wolf 1965) has been done in the field of partial coherence during recent years. The earliest papers on this subject are due to van Cittert (1936) and Zernike (1938) who have independently calculated the mutual intensity and degree of coherence for light from an extended, incoherent, quasi-monochromatic source. More recently, interest in the field of partial coherence has been concerned with image formation in partially coherent illumination (De 1955, Slansky 1955, Manzel 1958, Steel 1959, Slansky and Maréchal 1960), diffraction of partially coherent light by a plane aperture (Parrent and Skinner 1961), intensity interferometry (Hanbury Brown and Twiss 1956, 1957 a, b, Twiss and Hanbury Brown 1957, Rebka and Pound 1957, Mandel 1958, 1959, Twiss and Little 1959, Fano 1961), transient interference effects (Neugebauer 1962, Mandel 1962, Magyar and Mandel 1963), interference with partially coherent light (Thompson and Wolf 1957) and propagation of partially coherent light (Hopkins 1951, Wolf 1954, 1955, 1958, Parrent 1959). Pancharatnam (1956) has also studied the interference of two partially coherent beams in different states of polarization.

Born and Wolf (1962) and Wolf (1955, 1958) have studied the propagation of mutual coherence and expressed the mutual coherence at any two points in terms of the mutual coherence and its derivatives at two points on a surface enclosing these two points. We use this expression to study the intensity and the degree of coherence due to a partially coherent source. The partially coherent source is obtained by illuminating a narrow slit by a parallel thermal line source and the optical disturbances are taken to be scalar signals for the sake of simplicity.

In § 2 the mutual coherence at the slit, which is being illuminated by a thermal line source, is obtained. In § 3 we use the expression for the propagation of mutual coherence to obtain the mutual coherence at two points on the other side of the slit from the mutual coherence at the slit. In § 4 the variation of the intensity and the degree of coherence with different variables are studied.

2. Intensity and degree of coherence due to a line source

Let us consider an experimental arrangement in which a thermal line source of length $2L$ illuminates a parallel narrow slit of length $2a$ and of width d ($d \ll 2a$), the line joining the midpoints of the source and the slit being perpendicular to the source and to the plane of the slit. Let us divide the source into small elements of lengths dl_1, dl_2, ... situated around the points $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, ... which emit radiation independent of each other. If $V(\boldsymbol{y}, t)$ is the disturbance at the point $\boldsymbol{y}$ at time t, we have (taking $c = 1$)

$$V(\boldsymbol{y}, t) = \sum_m |\boldsymbol{y}-\boldsymbol{r}_m|^{-1} \int_0^\infty d\omega A(\boldsymbol{r}_m, \omega) \exp\{-i\omega(t-|\boldsymbol{y}-\boldsymbol{r}_m|)\} \tag{1}$$

where $A(\boldsymbol{r}_m, \omega)$ is the strength of the mth source in the frequency region near ω. The mutual coherence function $\Gamma(\boldsymbol{y}_1, \boldsymbol{y}_2; t_1, t_2)$, defined as

$$\Gamma(\boldsymbol{y}_1, \boldsymbol{y}_2; t_1, t_2) = \langle V^*(\boldsymbol{y}_1, t_1) V(\boldsymbol{y}_2, t_2) \rangle \tag{2}$$

in which the angular brackets denote an ensemble average, is given by

$$\Gamma(\boldsymbol{y}_1, \boldsymbol{y}_2; t_1, t_2) = \sum_{m,n} \Big[|\boldsymbol{y}_1-\boldsymbol{r}_m|^{-1} |\boldsymbol{y}_2-\boldsymbol{r}_n|^{-1} \int_0^\infty \int_0^\infty d\omega_1 \, d\omega_2 \langle A^*(\boldsymbol{r}_m, \omega_1) A(\boldsymbol{r}_n, \omega_2) \rangle$$
$$\times \exp\{i\omega_1(t_1-|\boldsymbol{y}_1-\boldsymbol{r}_m|) - i\omega_2(t_2-|\boldsymbol{y}_2-\boldsymbol{r}_n|)\} \Big]. \tag{3}$$

If the luminous intensity per unit length of the source is constant and is equal to I_0, we have, writing the intensity as $\langle V^* V \rangle$,

$$\langle A^*(\boldsymbol{r}_m, \omega_1) A(\boldsymbol{r}_n, \omega_2) \rangle = I_0 \delta_{mn} \delta(\omega_1-\omega_2) g(\omega_1) \Delta l_m \tag{4}$$

where $g(\omega_1)$ is the spectral distribution normalized to unity. On using equation (4) and changing the summation into an integration, equation (3) gives

$$\Gamma(\boldsymbol{y}_1, \boldsymbol{y}_2; t_1, t_2) = I_0 \int_{-L}^{L} dr |\boldsymbol{y}_1-\boldsymbol{r}|^{-1} |\boldsymbol{y}_2-\boldsymbol{r}|^{-1} \int_0^\infty d\omega \, g(\omega)$$
$$\times \exp\{i\omega(t_1-t_2-|\boldsymbol{y}_1-\boldsymbol{r}|+|\boldsymbol{y}_2-\boldsymbol{r}|)\} \tag{5}$$

where we write† $\boldsymbol{r}$ as $(r, 0, 0)$.

For points $\boldsymbol{y}_1 = (y_1, 0, S)$, $\boldsymbol{y}_2 = (y_2, 0, S)$ on the slit equation (6) gives, after integration over r,

$$\Gamma(\boldsymbol{y}_1, \boldsymbol{y}_2; t_1, t_2) = 2(I_0 L/S^2) \int_0^\infty d\omega \exp\{i\omega(t_1-t_2)\} g(\omega)$$
$$\times \frac{\sin\{\omega(y_1-y_2)L/S\}}{\omega(y_1-y_2)L/S} \tag{6}$$

if $\omega a^2/S$, $\omega L^4/S^3$, a^2/S^2 and L^2/S^2 are very much less than unity. In the quasi-monochromatic approximation (Pancharatnam 1963, Beran and Parrent 1964, p. 53), i.e. when

† We take the 1 axis along the source and the 3 axis along the line joining the midpoints of the source and the slit; the origin is at the centre of the source.

the function $g(\omega)$ is appreciable only for

$$\omega_0-\Delta\omega < \omega < \omega_0+\Delta\omega$$

($\Delta\omega/\omega_0$ being very much less than unity) and effectively zero outside this region, and

$$\left|t_1-t_2-\frac{L}{S}(y_1-y_2)\right| \ll \frac{1}{\Delta\omega}$$

the integrand in equation (7) can be replaced by

$$\exp\{\mathrm{i}\omega_0(t_1-t_2)\}g(\omega)\frac{\sin\{\omega_0(y_1-y_2)L/S\}}{\omega_0(y_1-y_2)L/S}. \tag{7}$$

Equation (6) then reduces to

$$\Gamma(\boldsymbol{y}_1, \boldsymbol{y}_2; t_1, t_2) = \frac{2I_0L}{S^2}\exp\{\mathrm{i}\omega_0(t_1-t_2)\}\frac{\sin\{\omega_0(y_1-y_2)L/S\}}{\omega_0(y_1-y_2)L/S}. \tag{8}$$

The degree of coherence

$$\gamma(\boldsymbol{y}_1, \boldsymbol{y}_2; t_1, t_2) = \frac{\Gamma(\boldsymbol{y}_1, \boldsymbol{y}_2; t_1, t_2)}{\{\Gamma(\boldsymbol{y}_1, \boldsymbol{y}_1; t_1, t_1)\Gamma(\boldsymbol{y}_2, \boldsymbol{y}_2; t_2, t_2)\}^{1/2}}$$

is then given by

$$\gamma(\boldsymbol{y}_1, \boldsymbol{y}_2; t_1, t_2) = \exp\{\mathrm{i}\omega_0(t_1-t_2)\}\frac{\sin\{\omega_0(y_1-y_2)L/S\}}{\omega_0(y_1-y_2)L/S}. \tag{9}$$

3. Intensity and degree of coherence due to the partially coherent slit

For points $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ in the field of the slit the mutual coherence $\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_2; t_1, t_2)$ obeys the equation

$$\left(\nabla_{x_1}{}^2-\frac{\partial^2}{\partial t_1{}^2}\right)\left(\nabla_{x_2}{}^2-\frac{\partial^2}{\partial t_2{}^2}\right)\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_2; t_1, t_2) = 0 \tag{10}$$

as this region does not contain any source. If we write the mutual coherence function as

$$\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_2; t_1, t_2) = \int_0^\infty\int_0^\infty \mathrm{d}\omega_1\,\mathrm{d}\omega_2\exp\{\mathrm{i}(\omega_1t_1-\omega_2t_2)\}\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_2; \omega_1, \omega_2) \tag{11}$$

the solution of equation (10) can be written as

$$\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_2; t_1, t_2) = \int_0^\infty\int_0^\infty \mathrm{d}\omega_1\,\mathrm{d}\omega_2\exp\{\mathrm{i}(\omega_1t_1-\omega_2t_2)\}\int_A\int \mathrm{d}^2y_1\,\mathrm{d}^2y_2$$
$$\times K^*(\boldsymbol{x}_1, \boldsymbol{y}_1, \omega_1)K(\boldsymbol{x}_2, \boldsymbol{y}_2, \omega_2)\Gamma(\boldsymbol{y}_1, \boldsymbol{y}_2; \omega_1, \omega_2) \tag{12}$$

where A is a surface which encloses the points $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ but not any source, and $K(x_i, y_i, \omega_i)$ is given by

$$K(\boldsymbol{x}_i, \boldsymbol{y}_i, \omega_i) = \frac{\partial}{\partial n_\mathrm{A}}G(\boldsymbol{x}_i, \boldsymbol{y}_i', \omega_i)|_{y_i'=y_i} \tag{13}$$

$\partial/\partial n_\mathrm{A}$ being the differentiation along the outward normal to the surface A and $G(\boldsymbol{x}_i, \boldsymbol{y}_i', \omega_i)$ being the Green function satisfying the equation

$$(\nabla_{x_i}{}^2+\omega_i{}^2)G(\boldsymbol{x}_i, \boldsymbol{y}_i', \omega_i) = \delta^3(\boldsymbol{x}_i-\boldsymbol{y}_i') \tag{14}$$

and the condition

$$G(\boldsymbol{x}_i, \boldsymbol{y}_i', \omega_i)|_{y_i'=y} = 0. \tag{15}$$

For the experimental set-up described in the previous section, $K(\boldsymbol{x}_i, \boldsymbol{y}_i, \omega_i)$ is given by (Beran and Parrent 1964, p. 40)

$$K(\boldsymbol{x}_i, \boldsymbol{y}_i, \omega_i) = \mathrm{i}(2\pi)^{-1}\omega_i|\boldsymbol{x}_i-\boldsymbol{y}_i|^{-1}\exp(\mathrm{i}\omega_i|\boldsymbol{x}_i-\boldsymbol{y}_i|) \tag{16}$$

if

$$|\boldsymbol{x}_i-\boldsymbol{y}_i| \gg 1/\omega_i$$

and

$$|\boldsymbol{n}\times(\boldsymbol{x}_i-\boldsymbol{y}_i)| \ll |\boldsymbol{x}_i-\boldsymbol{y}_i|$$

$\boldsymbol{n}$ being the unit vector perpendicular to the slit. On using equation (16), equation (12) reduces to

$$\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_2; t_1, t_2) = (2\pi)^{-2}\frac{\partial}{\partial t_1}\frac{\partial}{\partial t_2}\int_{\mathrm{A}}\int \mathrm{d}^2y_1\,\mathrm{d}^2y_2|\boldsymbol{x}_1-\boldsymbol{y}_1|^{-1}|\boldsymbol{x}_2-\boldsymbol{y}_2|^{-1}$$
$$\times\Gamma(\boldsymbol{y}_1, \boldsymbol{y}_2; t_1-|\boldsymbol{x}_1-\boldsymbol{y}_1|, t_2-|\boldsymbol{x}_2-\boldsymbol{y}_2|). \tag{17}$$

We wish to find the mutual coherence between two points in the field of the partially coherent slit. For this, we take A to be consisting of the plane of the slit and the surface at remote distances at which $V(\boldsymbol{y}, t)$ vanishes. For $\boldsymbol{x}_i = (x_i, 0, R)$, $i = 1, 2$, equations (17) and (8) give

$$\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_2; t_1, t_2) = 2(2\pi)^{-2}\omega_0{}^2\frac{I_0L}{S^2}\int_{\mathrm{A}}\int \mathrm{d}^2y_1\,\mathrm{d}^2y_2|\boldsymbol{x}_1-\boldsymbol{y}_1|^{-1}|\boldsymbol{x}_2-\boldsymbol{y}_2|^{-1}$$
$$\times\exp\{\mathrm{i}\omega_0(t_1-t_2-|\boldsymbol{x}_1-\boldsymbol{y}_1|+|\boldsymbol{x}_2-\boldsymbol{y}_2|)\}\,\frac{\sin\{\omega_0(y_1-y_2)L/S\}}{\omega_0(y_1-y_2)L/S}. \tag{18}$$

If x_1/R and x_2/R are very much less than unity, and only their leading terms are retained, equation (18) reduces to

$$\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_2; t_1, t_2) = 2(2\pi)^{-2}\omega_0{}^2\frac{I_0d^2L}{S^2R^2}\exp\left[\mathrm{i}\omega_0\left\{t_1-t_2-\frac{1}{R}(x_1{}^2-x_2{}^2)\right\}\right]$$
$$\times\int_{-a}^{a}\int_{-a}^{a}\mathrm{d}y_1\,\mathrm{d}y_2\exp\left\{\mathrm{i}\frac{\omega_0}{R}(x_1y_1-x_2y_2)\right\}\frac{\sin\{\omega_0(y_1-y_2)L/S\}}{\omega_0(y_1-y_2)L/S}. \tag{19}$$

On performing the integrations over y_1 and y_2, and introducing $\lambda_0 = 2\pi/\omega_0$, the wavelength corresponding to the mean frequency ω_0, we obtain

$$\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_2; t_1, t_2) = \frac{I_0d^2a}{2\pi\lambda_0SR^2}\exp\left[\frac{2\pi\mathrm{i}}{\lambda_0}\left\{t_1-t_2-\frac{1}{R}(x_1{}^2-x_2{}^2)\right\}\right]$$
$$\times\left(\frac{\cos 2\beta}{2\beta}\left[\ln\left|\frac{(\beta_1+\alpha)(\beta_2-\alpha)}{(\beta_1-\alpha)(\beta_2+\alpha)}\right|\right.\right.$$
$$-\mathrm{Ci}\{2(\beta_1+\alpha)\}+\mathrm{Ci}\{2(\beta_2+\alpha)\}+\mathrm{Ci}\{2(\beta_1-\alpha)\}$$
$$\left.-\mathrm{Ci}\{2(\beta_2-\alpha)\}\right]+\frac{\sin 2\beta}{2\beta}[\mathrm{Si}\{2(\beta_1+\alpha)\}$$
$$\left.+\mathrm{Si}\{2(\beta_2+\alpha)\}-\mathrm{Si}\{2(\beta_1-\alpha)\}-\mathrm{Si}\{2(\beta_2-\alpha)\}]\right) \tag{20}$$

where $\beta_1 = 2\pi ax_1/\lambda_0R$, $\beta_2 = 2\pi ax_2/\lambda_0R$, $2\beta = \beta_1-\beta_2$, $\alpha = 2\pi aL/\lambda_0S$ and $\mathrm{Ci}(x)$ and $\mathrm{Si}(x)$ are the functions defined by the equations

$$\mathrm{Ci}(x) = \int_{\infty}^{x}\mathrm{d}t\frac{\cos t}{t}, \qquad \mathrm{Si}(x) = \int_{0}^{x}\mathrm{d}t\,\frac{\sin t}{t}. \tag{21}$$

It should be remembered that in equation (21) I_0 is the luminous intensity per unit area of the source of length $2L$, $2a$ and d are the length and the width, respectively, of the slit A and S is the distance of the slit from the source.

To obtain the intensity, we put $x_1 = x_2 = x$, $t_1 = t_2 = t$ in equation (21); this gives

$$I(x) = \frac{I_0 d^2 a}{\pi\lambda_0 S R^2}\left[\frac{\sin^2(\beta_0-\alpha)}{\beta_0-\alpha} - \frac{\sin^2(\beta_0+\alpha)}{\beta_0+\alpha} + \mathrm{Si}\{2(\beta_0+\alpha)\} - \mathrm{Si}\{2(\beta_0-\alpha)\}\right] \tag{22}$$

where $\beta_0 = 2\pi a x/\lambda_0 R$. The degree of coherence is then given by

$$\begin{aligned}\gamma(x_1, x_2; t_1, t_2) = {} & \frac{1}{2}\exp\left[\frac{2\pi \mathrm{i}}{\lambda_0}\left\{t_1 - t_2 - \frac{1}{R}(x_1{}^2 - x_2{}^2)\right\}\right] \\ & \times\left(\frac{\cos 2\beta}{2\beta}\left[\ln\left|\frac{(\beta_1+\alpha)(\beta_2-\alpha)}{(\beta_1-\alpha)(\beta_2+\alpha)}\right|\right.\right. \\ & \left. - \mathrm{Ci}\{2(\beta_1+\alpha)\} + \mathrm{Ci}\{2(\beta_2+\alpha)\} + \mathrm{Ci}\{2(\beta_1-\alpha)\} - \mathrm{Ci}\{2(\beta_2-\alpha)\}\right] \\ & \left. + \frac{\sin 2\beta}{2\beta}[\mathrm{Si}\{2(\beta_1+\alpha)\} + \mathrm{Si}\{2(\beta_2+\alpha)\} - \mathrm{Si}\{2(\beta_1-\alpha)\} - \mathrm{Si}\{2(\beta_2-\alpha)\}]\right) \\ & \times\left(\left[\frac{\sin^2(\beta_1-\alpha)}{\beta_1-\alpha} - \frac{\sin^2(\beta_1+\alpha)}{\beta_1+\alpha} + \mathrm{Si}\{2(\beta_1+\alpha)\} - \mathrm{Si}\{2(\beta_1-\alpha)\}\right]\right. \\ & \left.\times\left[\frac{\sin^2(\beta_2-\alpha)}{\beta_2-\alpha} - \frac{\sin^2(\beta_2+\alpha)}{\beta_2+\alpha} + \mathrm{Si}\{2(\beta_2+\alpha)\} - \mathrm{Si}\{2(\beta_2-\alpha)\}\right]\right)^{-1/2}.\end{aligned} \tag{23}$$

4. Discussion

From equation (10) the degree of coherence at the slit can be written as

$$|\gamma(y_1, y_2; t_1, t_2)| = \left|\frac{\sin\{(y_1-y_2)/\epsilon\}}{(y_1-y_2)/\epsilon}\right| \tag{24}$$

where $\epsilon = \lambda_0 S/2\pi L$. Thus our experimental source, namely the slit A, contains coherent patches which extend over a length of order ϵ. We now consider two special cases: the limits $\epsilon \to \infty$ when the slit A is coherent, and $\epsilon \to 0$ when A is incoherent. In the coherent limit equations (23) and (24) give

$$I_{\mathrm{coh}}(x) = \frac{2\mathscr{I}_0 a^2 d^2}{\lambda_0 S^2 R^2}\frac{\sin^2\beta_0}{\beta_0{}^2} \tag{25}$$

and

$$|\gamma_{\mathrm{coh}}(x_1, x_2; t_1, t_2)| = 1 \tag{26}$$

where $\mathscr{I}_0 = 2I_0 L$ is the total luminous intensity of the source. In the incoherent limit we have

$$I_{\mathrm{incoh}}(x) = \frac{2I_0 a d^2}{\lambda_0 S R^2} \tag{27}$$

and

$$|\gamma_{\mathrm{incoh}}(x_1, x_2; t_1, t_2)| = \left|\frac{\sin\{2\pi a(x_1-x_2)/\lambda_0 R\}}{2\pi a(x_1-x_2)/\lambda_0 R}\right|. \tag{28}$$

Here we note the contrast that, while in the coherent limit $|\gamma_{\mathrm{coh}}|$ is always unity and the intensity $I(x)$ depends on x, the distance of the point $\boldsymbol{x}$ from the principal axis, in the incoherent limit it is $|\gamma_{\mathrm{incoh}}|$, which depends on the position of the points $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, and the intensity $I(x)$ is independent of x in our approximation. The variation of degree of coherence with $|\beta|$ is shown in figures 1 and 2 for $\alpha = 0{\cdot}5$, $1{\cdot}0$ and $\beta = \frac{1}{2}(\beta_1+\beta_2) = 0{\cdot}0$, $0{\cdot}5$.

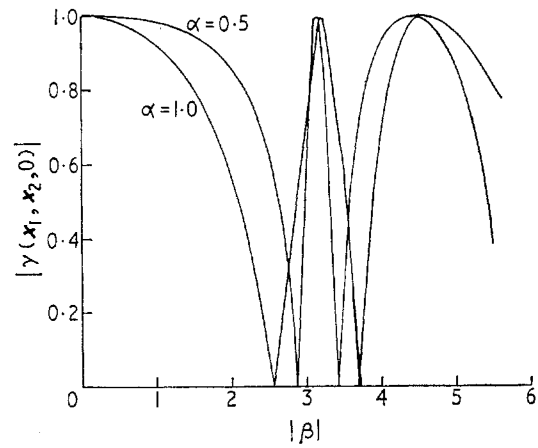

Figure 1. Variation of the degree of coherence $|\gamma(x_1, x_2, 0)|$ with $|\beta|$† for $\bar{\beta} = 0{\cdot}0$ and $\alpha = 0{\cdot}5, 1{\cdot}0$.

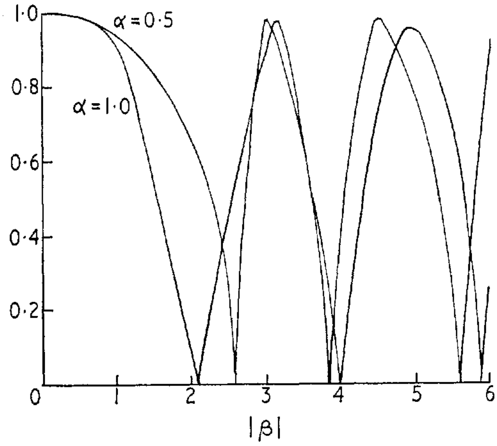

Figure 2. Variation of the degree of coherence $|\gamma(x_1, x_2, 0)|$ with $|\beta|$‡ for $\bar{\beta} = 0{\cdot}5$ and $\alpha = 0{\cdot}5, 1{\cdot}0$.

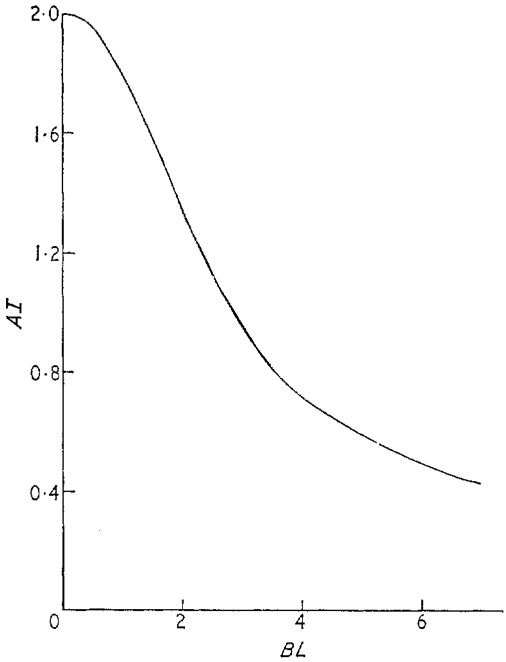

Figure 3. Variation of the intensity I with a/S. $A = \pi\lambda_0 R^2/2I_0 d^2$ and $B = 2\pi L/\lambda_0$.

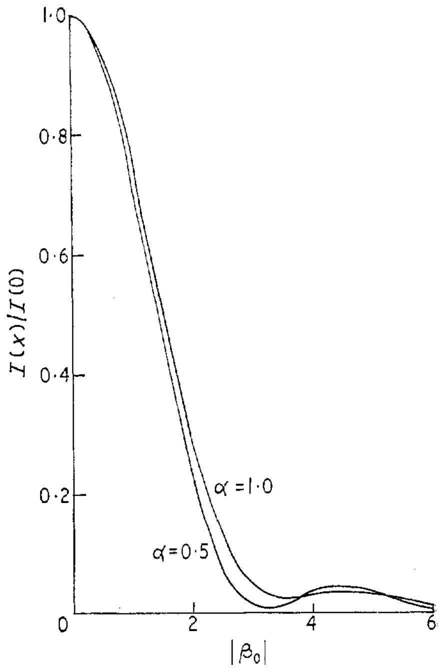

Figure 4. Variation of the intensity I with $|\beta_0|$ for $\alpha = 0{\cdot}5, 1{\cdot}0$.

† $\bar{\beta} = 0{\cdot}0$ corresponds to $x_1 + x_2 = 0$, i.e. the situation in which the two points are symmetrical about the principal axis. To obtain $\alpha = 0{\cdot}5$, one may take $S = 2{\cdot}00 \times 10^2$ cm, $L = 5{\cdot}00 \times 10^{-2}$ cm and $a = 1{\cdot}84 \times 10^{-2}$ cm with $\lambda_0 = 5790$ Å. To obtain $\alpha = 1{\cdot}0$, one may double L, the length of the source.

‡ $\bar{\beta} = 0{\cdot}5$ corresponds to $x_1 + x_2 = 0{\cdot}10$ cm for $R = 2{\cdot}00 \times 10^2$ cm and $a = 1{\cdot}84 \times 10^{-2}$ cm with $\lambda_0 = 5790$ Å.

From equation (23) we see that the intensity is symmetrical about the principal axis, i.e. $I(-x) = I(x)$. For $x = 0$, i.e. for points on the principal axis, equation (22) gives

$$I(x=0) = \frac{2\mathscr{I}_0 a^2 d^2}{\lambda_0{}^2 S^2 R^2}\left(\frac{\sin 2\alpha}{\alpha} - \frac{\sin^2\alpha}{\alpha^2}\right). \tag{27}$$

When a, d, S, R and the total luminous intensity of the source, $\mathscr{I}_0$ ($= 2I_0L$), are kept fixed, the intensity is a maximum at $L = 0$ (i.e. when the source is coherent; $\alpha = 0$ in equation (27)) and then it decreases as L increases. If I_0, d, L and R are fixed, by writing equation (27) as

$$I(x=0) = \frac{2I_0 d^2}{\pi\lambda_0 R^2}\frac{a}{S}\left\{\mathrm{Si}(2\alpha) - \frac{\sin^2\alpha}{\alpha}\right\} \tag{28}$$

we see that the intensity is zero at $a/S = 0$ and goes on increasing with increasing a/S. Variation of the intensity with a/S is shown in figure 3.

When $|\beta_0| \neq 0$, the intensity has a maximum or minimum when

$$\frac{\alpha}{|\beta_0|} = \frac{\tan\alpha}{\tan|\beta_0|}, \qquad \alpha \neq |\beta_0| \tag{29}$$

or

$$\frac{\alpha}{|\beta_0|} = \frac{\tan|\beta_0|}{\tan\alpha}, \qquad \alpha \neq |\beta_0|. \tag{30}$$

Maxima or minima occur according as $\tan|\beta_0|(\tan^2\alpha - \tan^2|\beta_0|)$ is greater or less than 1, respectively. The variation of intensity with $|\beta_0|$ for $\alpha = 0{\cdot}5$, $1{\cdot}0$ is shown in figure 4.

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